Perturbative Matching of Heavy-Light Currents with NRQCD Heavy Quarks

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We present further results for one-loop matching of heavy-light axial and vector currents between continuum QCD and a lattice theory with NRQCD heavy quarks and massless clover quarks.

1. Introduction

Lattice studies of hadronic matrix elements require matching between operators in full continuum QCD and those in the lattice theory being simulated. In the present article we will focus on an effective lattice theory which combines nonrelativistic (NRQCD) heavy quarks and clover light quarks. We wish to match the theories correct through $O(\frac{p}{M}\,,\,\alpha\frac{p}{M}\,,\,\alpha\, ap)$.

For heavy meson (e.g. B and B^*) leptonic and semileptonic decays, the relevant operators are heavy-light vector and axial vector currents, denoted in the continuum theory as $V_\mu \equiv \overline{q}\gamma_\mu h$ and $A_\mu \equiv \overline{q}\gamma_5\gamma_\mu h$. The first question that arises concerns the number and type of operators required in the effective theory.

For $\boxed{A_0 \text{ and } V_0}$ one finds 3 operators in the effective theory through $O(\frac{p}{M})$

$$J_t^{(0)} = \bar{q}(x)\,\Gamma_t\,Q(x), \qquad (\Gamma_t = \gamma_5\gamma_0 \text{ or } \gamma_0)$$

$$J_t^{(1)} = \tfrac{-1}{2M}\bar{q}(x)\,\Gamma_t\,\boldsymbol{\gamma\cdot\nabla}\,Q(x),$$
$$J_t^{(2)} = \tfrac{-1}{2M}\bar{q}(x)\,\boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\Gamma_t\,Q(x),$$

plus a dimension 4 discretization correction at $O(\alpha\, ap)$:

$$J_t^{disc} = -a\,\bar{q}(x)\,\boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\Gamma_t\,Q(x) = 2\,aM\,J_t^{(2)}$$

$$J_t^{(0)} \longrightarrow J_t^{(0),imp} = J_t^{(0)} + (\alpha\,\zeta_{disc}^{A_0/V_0})J_t^{disc}$$

(the spinors h and Q are related via a Foldy-Wouthuysen transformation).

For $\boxed{A_k \text{ and } V_k}$ one has 5 operators in the effective theory through $O(\frac{p}{M})$

$$J_k^{(0)} = \bar{q}(x)\,\Gamma_k\,Q(x), \qquad (\Gamma_k = \gamma_5\gamma_k \text{ or } \gamma_k)$$

$$J_k^{(1)} = \tfrac{-1}{2M}\bar{q}(x)\,\Gamma_k\,\boldsymbol{\gamma\cdot\nabla}\,Q(x),$$

$$J_k^{(2)} = \tfrac{-1}{2M}\bar{q}(x)\,\boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\Gamma_k\,Q(x),$$

$$J_k^{(3)} = \tfrac{-1}{2M}\bar{q}(x)\,\Gamma_s\nabla_k\,Q(x), \qquad (\Gamma_s = \gamma_5 \text{ or } \hat{I})$$

$$J_k^{(4)} = \tfrac{1}{2M}\bar{q}(x)\,\overleftarrow{\nabla}_k\Gamma_s\,Q(x).$$

Again there is an $O(\alpha\, ap)$ discretization correction to $J_k^{(0)}$. $J_k^{disc} = 2\,aM\,J_k^{(2)}$ and

$$J_k^{(0)} \longrightarrow J_k^{(0),imp} = J_k^{(0)} + (\alpha\,\zeta_{disc}^{A_k/V_k})J_k^{disc}$$

The desired relation between QCD hadronic matrix elements and nonperturbatively determined lattice current matrix elements is hence,

$$\langle\, A_0\,\rangle_{QCD} \;=\; \sum_{j=0}^{2} C_j^{A_0}\,\langle\, J_{A_0}^{(j)}\,\rangle_{LAT},$$

$$\langle\, V_k\,\rangle_{QCD} \;=\; \sum_{j=0}^{4} C_j^{V_k}\,\langle\, J_{V_k}^{(j)}\,\rangle_{LAT},$$

and similarly for V_0 and A_k.

The matching coefficients, C_j, have the following perturbative expansion,

$$C_j = 1 + \alpha\ \rho_j + O(\alpha^2) \qquad j = 0, 1$$
$$C_j = \alpha\ \rho_j + O(\alpha^2) \qquad j > 1$$

The goal is to calculate the ρ_j's.

2. Some Calculational Details

In the continuum theory we employ on-shell renormalization with naive dimensional regularization and the $\overline{MS}$ scheme. A gluon mass is introduced to handle IR divergences that eventually cancel between continuum and the lattice. The light quark mass is set equal to zero. On the lattice we worked mainly with an NRQCD action correct through $O(p/M)$. Results with higher order relativistic corrections also exist. The lattice light quarks are massless clover fermions. The light and heavy quark actions and the lattice current operators are all tadpole improved.

3. Results for One-Loop Coefficients

In figures 1. and 2. we plot the one-loop coefficients ρ_0 and $\rho_j/(2aM)$, $j > 0$, for all four current types, V_k, A_k, A_0, and V_0 versus the inverse dimensionless bare heavy quark mass [1]. The "bursts" show ρ_0 with $log(aM)$ fixed to $log(2)$, where the choice of $aM = 2$ is somewhat arbitary, corresponding to the value of the bare dimensionless heavy quark mass appropriate for the physical b-quark on $a^{-1} \approx$ 2GeV lattices. With the logarithmic dependence taken out one can compare with ρ_0 for the static theory, as indicated for V_k and A_0[2]. One also sees that $\rho_2/(2aM)$ (the "diamonds") goes to a non-vanishing value as $aM \to \infty$. This is due to the mixing with the discretization correction, J_μ^{disc}.

4. Applications

Matching coefficients presented here have already been applied to several studies of B and B^* meson decay constants [3–6]. In Fig.3 we show results for the vector meson decay constant $a^{3/2} f_V \sqrt{M_V}$ on dynamical HEMCGC configurations[5]. The squares represent tree-level and the crosses and diamonds one-loop results using $\alpha_V(q^*)$ with $q^* = \pi/a$ and $q^* = 1/a$ respectively. The physical B^* meson corresponds to $1/aM_0 \approx 0.5$. The one-loop correction is a 13 $\sim$ 21% effect depending on q^*. This is a slightly larger decrease than is found for the pseudoscalar decay constant based on A_0.

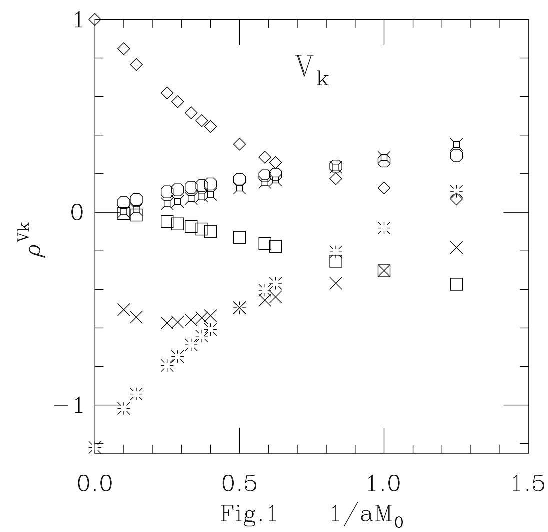

Figure 1. One-loop Coefficients ρ_j. crosses : ρ_0 squares : $\rho_1/2aM$; diamonds : $\rho_2/2aM$ (includes ζ_{disc}) ; octagons : $\rho_3/2aM$; fancy squares : $\rho_4/2aM$; bursts : ρ_0 with $log(aM)$ fixed at $log(2)$.

Acknowledgements : This work was supported by DOE Grants DE-FG03-90ER40546 and DE-FG02-91ER40690 and by a NATO grant CRG 941259.

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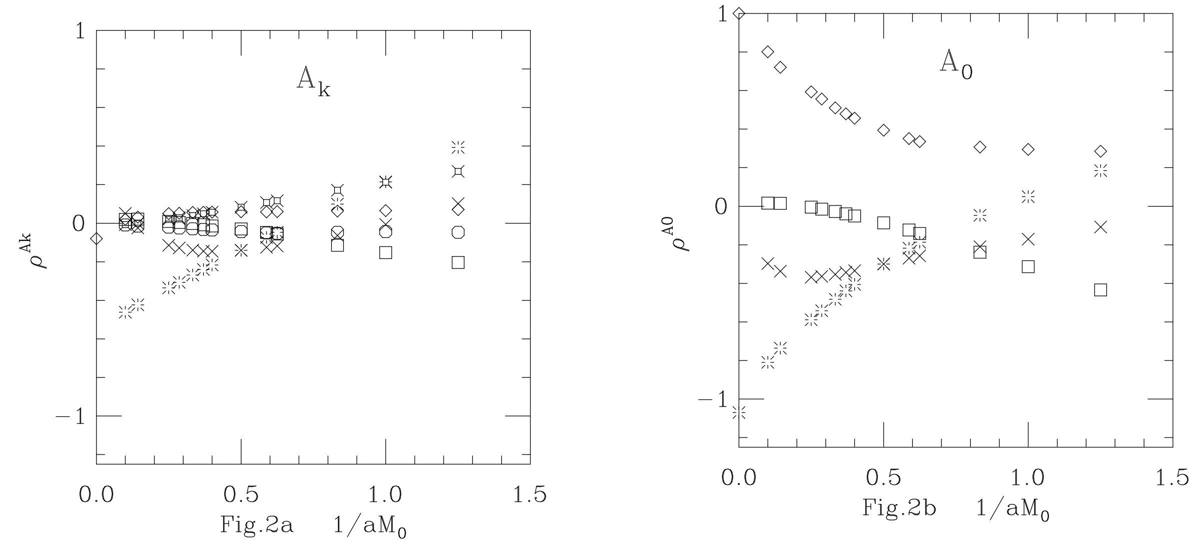

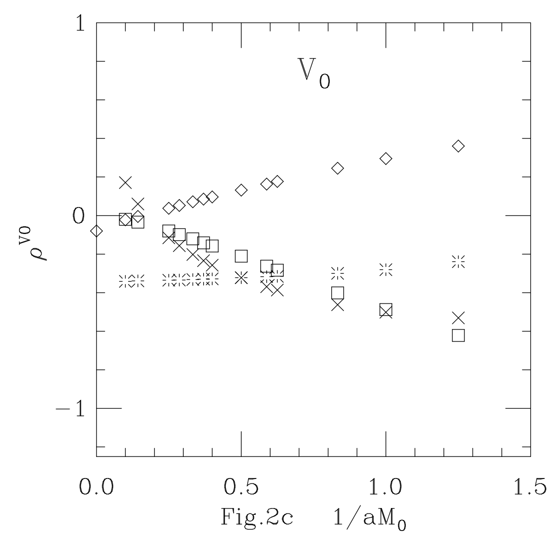

Figure 2. Same as Fig.1 for A_k, A_0 and V_0.

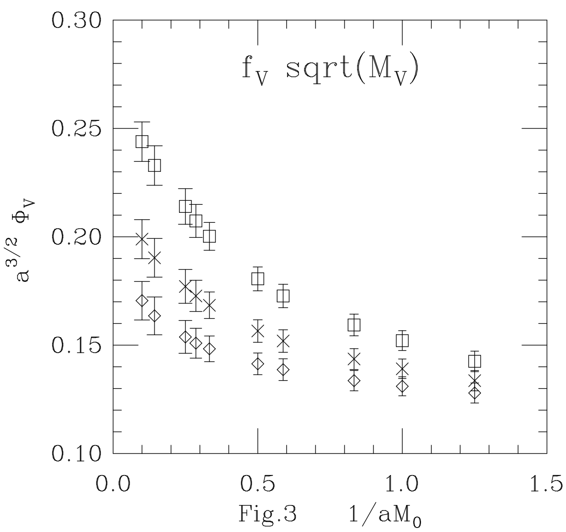

Figure 3. squares : tree-level ; crosses : $\alpha_V(\pi/a)$ diamonds : $\alpha_V(1/a)$.